Mathematical modeling and optimization cycle
gas turbine cogeneration

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Abstract—combined heat and power (CHP) utilization is an essential and inevitable because the conventional methods usage are inefficient and pollutant. The cogeneration has economic and environmental advantages. In this paper, a thermodynamic model was developed to analyze the performance of cogeneration plant based on ideal brayton cycle. This gas turbine cycle modeling includes compressor, turbine, combustion chamber and heat recovery steam generator (HRSG). The efficiency of cycle supposed as objective function and outlet temperature of combustion chamber, payback time and the ratio of heat to power generation supposed as constraints. By analyzing this model, the dependency of efficiency to pressure ratio, turbine outlet temperature and chimney inlet temperature has been studied and the optimal value of the objective function and each of the above factors have been obtained.

Keywords—cogeneration cycle, gas turbine

I. INTRODUCTION

USUALLY, industries, commercial buildings and residential buildings power requirements supplied by power plant whereas their heat requirements produced by themselves utilities. Another way to supply heat and power is the simultaneous production of electricity and useful heating by one system [1].

The use of gas turbines for generating electricity dates back to 1939. Today, gas turbines are one of the most widely-used power generating technologies. Gas turbines are a type of internal combustion (IC) engine in which burning of an air-fuel mixture produces hot gases that spin a turbine to produce power. It is the production of hot gas during fuel combustion, not the fuel itself that gives gas turbines the name. Gas turbines can utilize a variety of fuels, including natural gas, fuel oils, and synthetic fuels. Combustion occurs continuously in gas turbines, as opposed to reciprocating IC engines, in which combustion occurs intermittently [2].

Generally, gas turbines are used extensively by the oil and gas industries to actuate pumps and compressors and power generation. By using the turbine discharged hot gas, simultaneously; gas turbine generates power and useful heat [3].

II. THERMODYNAMIC MODEL

Using gas turbine with heat recovery (HRSG) is one of the methods for simultaneous production of electricity and heat that can be modeled as a brayton cycle. Based on Fig.1 during an isentropic process in compressor, inlet air compressed up to point 2 and arrived to combustion chamber and during a constant pressure process in combustion chamber, fluid heated as \( Q_H \) and reaches to point 3. At this point, fluid enters to turbine and during an isentropic expansion power produced and exhausted fluid from turbine at point 4 arrived to heat recovery (HRSG). In HRSG, exhaust air from the turbine is taken through a constant pressure process and transferred to another fluid (water or air) and air exits at point 5 and goes to chimney.

Fig. 1 gas turbine cycle with HRSG

The concept of efficiency that used for cycle performance investigating can be defined as:” The ratio of useful output
energy to total input energy." In this cycle, useful output energy includes turbine output power and heat transferred in HRSG. Also, input energy is transferred heat in combustion chamber \((Q_H)\).

For cycle optimization the efficiency should be increased. Some constraints prevent from increasing of efficiency. Due to the issues mentioned above, the efficiency of cycle that is objective function obtained as follows:

\[
\eta = \frac{\dot{W}_T}{Q_H} - \frac{\dot{W}_C}{Q_L}
\]

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\]

And the dimensionless parameters are defined as follows:

\[
\alpha_{HR} = \frac{T_4}{T_1}
\]

\[
\theta_c = \frac{T_3}{T_1}\left(\frac{p_2}{p_1}\right)^\frac{\gamma - 1}{\gamma}
\]

\[
\alpha_{eT} = \frac{T_4}{T_1}
\]

As a result, the efficiency can be written as follows:

\[
\eta_{th} = \left(\frac{Q_H - Q_L}{Q_H}\right)\left(1 - \frac{Q_L}{Q_H}\right)
\]

\[
1 - \left[\frac{T_1(T_5/T_1 - 1)}{T_2(T_5/T_2 - 1)}\right]_{T_1/T_2 = T_1/T_2}
\]

\[
\eta_{th} = 1 - \left[\frac{(\alpha_{HR} - 1)}{\theta_c} (\alpha_{eT} - 1)\right]
\]

\[
\min : f = \left[\frac{(\alpha_{HR} - 1)}{\theta_c} (\alpha_{eT} - 1)\right] - 1
\]

The constraints that cycle (this issue) is facing with them as follows:

1) The turbine inlet temperature \((T_3)\) should be less than 700°C because above this temperature corrosion problems will be created.
2) The ratio of power to heat generation should be less than 5.
3) Payback time will be 10 years. In this paper, SGT-300, the product of SIEMENS Company, is gas turbine for cogeneration. According to its catalog, below information about SGT-300 is used for modeling [4].

Considerations:

System works continuously.
Outlet flow gas from turbine is 30.2 kg/s [4].
Estimated price for 1000 kJ heating is 0.07 $. Approximately, the installation expenditure for a CHP unit (gas turbine and HRSG) is 1000 $ per KWe. As for the capacity of gas turbine, its installation expenditure will be 7.9 million dollars [3].
4) The HRSG according to heat price, turbine expenditure and payback time should be recovering 3000 kJ heat per second. In this paper the efficiency of HRSG is assumed 85%. The equations related to the rate of heat recovery as follows:

\[
0.85\dot{m}_c(T_4 - T_3) \geq 3000 \rightarrow
\]

\[
0.85\dot{m}_c(T_1(\alpha_{eT} - \alpha_{HR}) \geq 3000
\]

\[
4.65 + \alpha_{HR} - \alpha_{eT} \leq 0
\]

5) Pressure ratio should be less than 14.2 and more than 11:

\[
P_2/P_1 \geq 11, P_2/P_1 \leq 14.2, \theta_c = (P_2/P_1)^\frac{\gamma - 1}{\gamma}, n_{air} = 0.28 \rightarrow
\]

\[
2 \leq \theta_c \leq 2.1
\]

6) Because this turbine is an industrial turbine and its heat recovery used for industrial process, minimum temperature of HRSG outlet should be more than 100°C.

\[
T_5 \geq 100 \rightarrow \alpha_{HR} \geq 4
\]

7) Turbine outlet temperature should be more than 200°C.

\[
T_4 \geq 200 \rightarrow \alpha_{eT} \geq 8
\]

Objective function and constraints obtained as follows:

Max : \[\eta_{th} = 1 - [\theta_c (\alpha_{eT} - 1)]\]

Min : \[f = [(\alpha_{HR} - 1)/(\theta_c) (\alpha_{eT} - 1)] - 1\]

subject :

\[
\alpha_{eT} \theta_c - 28 \leq 0
\]

\[
\left[\theta_c - 1(\alpha_{eT} - 1)/(\alpha_{eT} - \alpha_{HR})\right] - 5 \leq 0
\]

\[
4.65 + \alpha_{HR} - \alpha_{eT} \leq 0
\]

\[
\alpha_{HR} \geq 4 \rightarrow 4 - \alpha_{HR} \leq 0
\]

\[
\theta_c \geq 2 \rightarrow 2 - \theta_c \leq 0
\]

\[
\alpha_{eT} \geq 8 \rightarrow 8 - \alpha_{eT} \leq 0
\]

III. ANALYSIS

Optimum value for efficiency and other independent parameter obtained with analysis the equation (19). This model has been solved by Matlab and the optimal value of objective function is -0.8132586. It means that the efficiency of cycle is 81.32 percent. As for regardless of irreversibility, leakage and pressure drops is rational and logistic. Other optimal parameters as follows:
\( \alpha_{HR} = 4 \)

\[
X_{opt} \Rightarrow \theta_C = 2.1 \quad (20)
\]

\( \alpha_{ef} = 8.65 \)

According to equation (20), HRSG outlet temperature \((T_5)\) is 100°C and pressure ratio is 14.2 and the temperature of air that enters to HRSG \((T_4)\) is 243°C.

IV. CONCLUSION

In this paper, supposed the gas turbine has been exists and by adding HRSG, gas turbine converted to a CHP system. The thermodynamic model of this system is optimized by mathematical approaches. For cycle optimization the efficiency should be increased. Some constraints prevent from increasing of efficiency. Turbine inlet temperature, ratio of power to heat generation and payback time are important constraints.

Although this thermodynamic model is an ideal model but its results can be used for real cases. Result showed by raising turbine inlet temperature, the efficiency and payback time increased. In higher temperature turbine must be produced from special and expensive material. Expensive material increased turbine expenditure and payback time. Increasing in pressure ratio will be same as increasing turbine inlet temperature.

HRSG is constrained by minimum exhaust temperature, HRSG efficiency, heat requirement and heat prices.

REFERENCES